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TEACHING OF THE PRINCIPLES FOR HANDLING PHYSICAL QUANTITIES IN CHEMISTRY EDUCATION

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Summary. This contribution firstly suggests three principles for handling physical quantities from quantity calculus, and secondly deals with how to teach them to students. Saying “let p be the atmospheric pressure” is based on the principle of unit-independence. Scientific expressions such as $p = 1013.25$ hPa can be derived from the principle of measurement. Equating two physical quantities is founded on the principle of quantity balance. Students could overcome the difficulty in learning quantity calculus if they practiced step by step the three principles.

Introduction

Quantity calculus, i.e., a method of handling physical quantities [1], is so fundamental in modern science that it needs to be taught in chemistry education. The origin of quantity calculus can be traced back to the late nineteenth century [2]. Guggenheim [3] reported that the notation of quantity calculus is very useful, and that its use was spreading, but surprisingly slowly. White [4] summarized the brief history of quantity calculus and also said that little has changed with respect to this situation. What is the difficulty in teaching and learning quantity

calculus? An effective way to teach students an object shows this concretely and illustrates it clearly. One has to analyze the essentials of quantity calculus in order to answer the question. This contribution firstly extracts three principles from quantity calculus, and secondly shows how to teach them to students.

A standard book [1] expresses, for example, the atmospheric pressure p as $p = 1 \text{ atm}$, $p = 1013.25 \text{ hPa}$, $p = 1.01325 \text{ bar}$ and $p = 760 \text{ mmHg}$, i.e.,

$$p = 1 \text{ atm} = 1013.25 \text{ hPa} = 1.01325 \text{ bar} = 760 \text{ mmHg}$$

This example suggests that there are two principles for manipulating physical quantities in quantity calculus and both are concerned with the notation of physical quantities. “Each symbol denotes a physical quantity, not its measure in terms of particular units; it is therefore sometimes called quantity calculus [3].” This statement is referred to as the principle of unit-independence of physical quantities, or simply the principle of unit-independence. Saying “let p be the atmospheric pressure” and “let v be the volume of solution” is based on the principle of unit-independence.

Everyone can find scientific symbols, for example, 200 mL on the labels of milk bottles, in everyday life. The second principle for handling physical quantities is concerned with such scientific expressions. Henderson [5] stated, “Let each symbol in a problem not only represent the quantity numerically, but also contain the units.” Scientific expressions such as $p = 1013.25 \text{ hPa}$ and $v = 200 \text{ cm}^3$ (= 200 mL) lead to a general form that is made up of a string of Arabic numerals, a space and a combination of units. The following discussion shows that such a scientific expression can be derived from the principle of measurement of physical quantities, or simply from the principle of measurement, and that the space between a string of Arabic numerals and a combination of units denotes the multiplication sign. If the atmospheric pressure p is measured in terms of the unit atm (hPa, bar, and mmHg), then one has $p = 1 \text{ atm}$ ($p = 1013.25 \text{ hPa}$, $p = 1.01325 \text{ bar}$ and $p = 760 \text{ mmHg}$).

Problem 1 for student practice [6] is: What is the mass of salt in 10 cm^3 of a solution of 2 g/dm^3 ? Answering this problem needs the third principle for handling physical quantities, called hereafter the principle of balancing of physical quantities, or simply the principle of quantity balance. It seems that students could overcome the difficulty in learning quantity calculus if they practiced step by step the above-mentioned principles. The present paper points out that the key concept for successful understanding of the three principles is ‘dimension’.

Dimension as a class of physical quantities

In the real world there are many physical quantities such as length, volume, mass, time, pressure, concentration, amount of substance and so on. Let us com-

pare one physical quantity with another and get the quotient between them in a pair of physical and/or chemical systems. There is a variety of methods for such comparison in the two systems. It is possible to compare, for example, the mass of a substance and that of a piece of metal using a top-balance, but it is impossible to compare the mass and the volume of milk bottles. After the successive operations a set of physical quantities in question is classified into classes (subsets). Such a class is called a dimension in physical sciences [7]. In other words, one can say that two physical quantities, if the comparison between them is possible, belong to the same dimension; inversely, it is possible to compare two physical quantities belonging to the same dimension. A dimension name is denoted hereafter by use of square brackets; e.g., [length], [volume], [mass], [time], [pressure], [concentration] and [amount of substance].

Let us treat [mass] as an example. One passes through one's mind a collection of masses of given physical/chemical systems; i.e., [mass] stands for the collection. This collection includes the international prototype of the kilogram, a weight of 1/1000 of the international prototype, the mass of an object (e.g., salt in a solution) which one wants to measure, etc. A pair of curl brackets indicates such an assembly. One can then write this situation as

$$[\text{mass}] = \{ \text{mass of the prototype, mass of a weight,} \\ \text{mass of salt in a solution, ...} \}$$

Expression of physical quantities as a representative

Each of the letters, x , a and b , in algebraic equations such as $x^2 + ax + b = 0$, represents an element belonging to the class of real numbers; and then in mathematics an element chosen in a class is called its representative. In dimensions one can similarly treat each physical quantity as a representative.

The symbol kg is recommended for the mass of the international prototype, and the mass of 1/1000 of kg is denoted by g [1]. Let m be the mass of a physical/chemical system, say, of salt in a solution. The dimension (collection) of masses is now rewritten in symbolic form as

$$[\text{mass}] = \{ \text{kg, g, } m, \dots \}$$

Here the symbol of each unit represents a physical quantity, and m is independent of the choice of units. The name 'mass' means the mass of a system on a certain occasion, and the name of a dimension on another occasion. Note that, generally speaking, a physical unit and the notation cannot be determined by use of physical / chemical rules in nature, but are chosen conveniently by the human being.

The mass m of salt in a solution is equal to the product of its concentration c and the volume v ; this is usually written as a formalized equation $m = c \times v$, which is relating to [mass] and two dimensions,

$$[\text{concentration}] = \{ \text{g/dm}^3, c, \dots \} \text{ and } [\text{volume}] = \{ \text{dm}^3, \text{cm}^3, v, \dots \}$$

It should be noted that each physical quantity in the formalized equation $m/v = c$ is based on the principle of unit-independence. Students can easily learn such formalized equations under the understanding that a physical quantity is a representative in the corresponding dimension.

Measurement of physical quantities in terms of units

A measurement of physical quantities compares one physical quantity with another of the same kind in two physical and/or chemical systems, and determines the quotient between them. Here one of the two physical quantities, selected as a standard of reference, is called the unit [8]. After a measurement of a physical quantity in terms of the unit, one thus obtains a numerical value; i.e., a physical quantity divided by the unit is equal to a numerical value. The following is thus a general expression:

$$\text{physical quantity} = \text{numerical value} \times \text{unit}$$

This is hereafter cited as $q = a \times u$, which means that the choice of a physical quantity and a unit makes measurement possible, and determines the numerical value. Such a numerical value is called a measure of the physical quantity by Guggenheim [7]. The expression $q/u = a$, or $q = a \times u$, is based on the principle of measurement in quantity calculus. Let us measure the mass m of 1 dm³ of a solution in terms of the unit g in [mass]. The quotient between the mass m and the unit g is then equal to, say 2; i.e., $m/g = 2$, or $m = 2 \times g = 2 \text{ g}$; similarly, one has $\text{kg/g} = 1000$, or $\text{kg} = 1000 \times \text{g} = 1000 \text{ g}$.

The phrase '10 cm³ of a 2 g/dm³ solution' is given in Problem 1. This means that the volume of a portion is measured in terms of the unit cm³ in [volume], and one gets the quotient between them; hence, $v(\text{portion}) = 10 \times \text{cm}^3 = 10 \text{ cm}^3$. One knows similarly $c(\text{whole}) = 2 \text{ g/dm}^3$ in [concentration]. The generalized expression ' $a u$ ' represents a physical quantity, and the space in the expression indicates the multiplication sign.

Using the principles in problem-solving

Problem 1 well illustrates the roles of dimensions, their representatives and the three principles in quantity calculus. Let m be the unknown mass of salt. Note that it should be unnecessary to state the mass in units. This first step uses

the principle of unit-independence. Anyone who can denote an entry of dimension [mass] as m , could solve the problem easily; the unknown parameter m is just a representative in dimension [mass].

The second step in problem solving uses the principle of measurement. The concentration of 10 cm^3 is then able to be expressed as $c(\text{portion}) = m/(10 \text{ cm}^3)$; and one knows the concentration for the whole solution as $c(\text{whole}) = 2 \text{ g/dm}^3$. Listing the data in the form $q = a \times u$ is based on the principle of measurement.

The next step uses the principle of quantity balance. One has to search for the physical quantity for equating (PQE) in the list of $q = a \times u$. Intensive physical quantities such as concentration can be applied to two systems of different size. The scientific equation, $c(\text{portion}) = c(\text{whole})$, in which PQE is concentration, is then obtained; this process reminds us of an alternative name to quantity calculus, 'equations of quantities [3]'. Understanding that there are both $c(\text{portion})$ and $c(\text{whole})$ in the same dimension [concentration] makes it possible for students to balance the two physical quantities. One now has an equation $m/(10 \text{ cm}^3) = 2 \text{ g/dm}^3$ with the unknown physical quantity m .

The last step uses the alternation of PQE based on the principle of quantity balance. Multiplying both sides of the equation $m/(10 \text{ cm}^3) = 2 \text{ g/dm}^3$ by (10 cm^3) leads to

$$(10 \text{ cm}^3) m/(10 \text{ cm}^3) = (10 \text{ cm}^3)(2 \text{ g/dm}^3)$$

in which PQE changes from concentration to mass, because $m = c \times v$. One has thus the answer $m = 20 \text{ mg}$ by canceling. Note that the physical meaning of the equalities such as $m/v = c$, $c(\text{portion}) = c(\text{whole})$ and $m/(10 \text{ cm}^3) = 2 \text{ g/dm}^3$ in the above-mentioned discussion is clear because each equation satisfies the principle of quantity balance.

Problem 1 can be read as Problem 2 in a form that is closely connected with chemistry: What is the amount of substance in 10 cm^3 of a 2 mol/dm^3 solution? The unknown mass m in Problem 1 is replaced by the unknown amount of substance n in Problem 2.

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ОПЕРИРАНЕ С ФИЗИЧНИТЕ ВЕЛИЧИНИ В ОБУЧЕНИЕТО ПО ХИМИЯ

Физичните величини в обучението по химия могат да бъдат разглеждани от гледна точка на три принципа с оглед пресмятанията с тях и начините на тяхното преподаване. Твърдението “нека p е атмосферното налягане” се базира на *принципа на независимост от единиците (unit-independence)*. Изразът “ $p=1013.25 \text{ hPa}$ ” следва *принципа на измерването (principle of measurement)*. Сравняването на две физични величини се основава на *принципа на приравняване на величините (quantity balance)*. Учениците могат да преодолеят трудностите при пресмятанията с величини, ако стъпка по стъпка последователно следват горните принципи.

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